

Mixing-Induced  $CP$  Violation in  $B \rightarrow P_1 P_2 \gamma$  in Search of Clean New Physics Signals

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We show that in a decay of the form  $B_d$  or  $B_s \rightarrow P_1 P_2 \gamma$  (where  $P_1$  and  $P_2$  are pseudoscalar mesons), through a flavor changing dipole transition, time dependent oscillations can reveal the presence of physics beyond the Standard Model. If  $P_1$  and  $P_2$  are  $CP$  eigenstates (*e.g.* as in  $B_d \rightarrow K_S \pi^0 \gamma$ ), then to leading order in the effective Hamiltonian, the oscillation is independent of the resonance structure. Thus data from resonances as well as from nonresonant decays can be included. This may significantly enhance the sensitivity to new physics of the method. If  $P_1$  is a charged particle, and  $P_2$  its anti-particle (*e.g.* as in  $B_d \rightarrow \pi^+ \pi^- \gamma$ ), one has the additional advantage that both the magnitude and the weak phase of any new physics contribution can be determined from a study of the angular distribution. These signals offer excellent ways to detect new physics because they are suppressed in the Standard Model. We also show that the potential contamination of these signals originating from the Standard Model annihilation diagram gives rise to photons with, to a very good approximation, the same helicity as the dominant penguin graph and thus causes no serious difficulty. The formalism which applies to the case where  $P_1$  and  $P_2$  are  $C$  eigenstates also further generalizes to the case of final states containing multiple  $C$  eigenstates and a photon. This suggests several additional channels to search for new physics, such as  $K_S \eta'(\eta) \gamma$ ,  $\phi K_S \gamma$  *etc.* We also emphasize that the contribution of non-dipole interactions can be monitored by the dependence of the mixing-induced  $CP$  asymmetry of non-resonant modes on the Dalitz variables. Furthermore, using a number of different final states can also provide important information on the contribution from non-dipole effects.

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## I. INTRODUCTION

The oscillation of neutral ground state mesons has proved to be a sensitive probe of  $CP$  violation and thus a sensitive probe for physics at an energy scale well beyond the mass of the meson itself. In recent years the oscillation of the neutral  $B_d$  meson produced at  $B$  factories or hadronic  $B$  experiments has provided a means to test the hypothesis that the Standard Model (SM) generates  $CP$  violation [1] through the Cabibbo-Kobayashi-Maskawa (CKM) mechanism [2]. In this approach new physics (NP) becomes evident if the CKM interpretation cannot consistently explain the results.

Within the realm of  $B$  physics, radiative decays resulting from the quark level transitions  $b \rightarrow q \gamma$ , where  $q = d$  or  $s$ , have long been recognized [3] as very good channels to look for NP. Indeed the experimental effort [4] to measure the rate of  $b \rightarrow s \gamma$  both in exclusive and inclusive channels has reached the point where the comparison with the SM is dominated by theoretical errors. Further reduction in the theory errors appears rather difficult. Clearly then it is advantageous to use in addition to the rates other observables that can reveal new physics.

One such well known observable is the direct  $CP$  partial rate asymmetry. The predicted SM asymmetry [5] for  $B_d \rightarrow X_s \gamma$  is rather small, about 0.5% and the experimental bound [4] is about a factor of ten above that. For  $B_d \rightarrow X_d \gamma$  the predicted asymmetry in the SM is actually quite large (around 15%) [5] but its branching ratio is rather small and also it is experimentally more challenging due to higher backgrounds. Nonetheless, this is clearly a promising technique in which efforts are currently being directed.

Another approach was suggested in [6] (AGS), and has been recently implemented experimentally [7], where the oscillations of the neutral  $B$  mesons are exploited. Such oscillations in the radiative decays of neutral  $B$  mesons are suppressed in the SM so if a significant signal of this sort is detected then NP is directly established. Indeed the cleanliness of the NP signal in the AGS technique provides one of the important motivations for a “super  $B$  factory” [8].

The key is the fact that the photon is polarized; the short distance Hamiltonian of the SM makes the very special prediction that photons from  $b$  ( $\bar{b}$ ) decay are dominantly left (right) polarized with the same weak phase. Thus any method such as AGS which measures an observable that vanishes with the SM polarized photons is a null test for new physics with little dependence on theoretical uncertainties concerning the hadronization of the final state. Recently it was pointed out [9] that decays of the form  $B^0 \rightarrow P^0 P^0 \gamma$ , where  $P^0$  is any spin-0  $CP$  eigenstate, can be used to probe AGS oscillations in  $b \rightarrow d\gamma$  transitions.

The problem with photons from  $B$  decay is, of course, that their polarization is not easily measured with current detector technology and so indirect methods must be used to probe polarization dependent observables as is the case in AGS. Besides AGS some other methods have been suggested to learn about the photon polarization. Consider, for example in the case of  $b \rightarrow s\gamma$ , mesonic decays of the form  $B_d \rightarrow X_s \gamma$ . Because the initial state has total angular momentum  $J = 0$  the helicity of the photon must be the same as the net helicity of the  $X_s$ . In [10, 11] (GP) the photon polarization is probed by considering the interferences of various  $K\pi\pi$  resonances. This approach has an advantage that with the four body final state, parity odd observables may be constructed but has a disadvantage that the interpretation of the angular distributions requires some understanding of the  $1^-$ ,  $1^+$  and  $2^+$  kaonic resonances. Another mode which may be useful to measure the photon polarization in  $b \rightarrow s\gamma$  transitions is the recently discovered  $B \rightarrow \phi K \gamma$  [12]. The photon polarization may also be studied in  $\Lambda_b$  decays [11, 13].

Another approach suggested in [14, 15] is to “resolve” the photon to an  $e^+e^-$  pair. This may either be accomplished via interaction of the photon with matter through a Bethe-Heitler conversion or internally where the photon is virtual. Measuring photon polarization through the Bethe-Heitler conversion may prove experimentally challenging but would be a method of general utility. Furthermore, the detection of such  $e^+e^-$  pairs in the vertex detector at a  $B$  factory experiment would enable determination of  $B$  meson decay positions in decay modes containing only photons in the final state. However, since the amount of material in the inner detector must be kept to a minimum in order to improve the experimental resolution, the rate of such conversion is rather low. Internal conversion would be present and dominate over short distance  $e^+e^-$  production at low electron-positron invariant mass. The event rate would, of course, be smaller than that for direct photons. An additional feature of this channel would be the ability to study the  $CP$  properties of the short distance electron positron pairs produced at a larger invariant mass [16].

In this paper, we will consider a variation on the processes considered in [6] where we will assume that the final state consists of two pseudoscalar mesons and a hard photon. For oscillations to occur, the mesons are required to be eigenstates of charge conjugation; since they have definite parity, they are also  $CP$  eigenstates. The use of a hard photon offers the dual advantage that (1)  $CP$  violation due to SM bremsstrahlung (discussed in [17]) is suppressed and (2) experimental backgrounds are suppressed at higher photon energy.

Of course, a final state such as  $K_S \pi^0 \gamma$  is less general than the  $K\pi\pi\gamma$  states considered in [10], however, in this case a simple analysis can lead to powerful conclusions regarding new physics. Indeed, in the case of  $B_d \rightarrow K_S \pi^0 \gamma$ , an initial study may be carried out as an extension of the existing analyses for the decay  $B_d \rightarrow K^{*0} \gamma$ ,  $K^{*0} \rightarrow K_S \pi^0$  [7], as our discussion shows that all  $B_d \rightarrow K_S \pi^0 \gamma$  decays, not only those produced via the  $K^{*0}$  resonance, may be included. By using such a generalized final state, not only it is possible to increase the statistics, but one can also extract useful information regarding the possible contributions to  $H_{\text{eff}}$  from operators other than the Standard Model dipole. A very important characteristic of the mixing-induced  $CP$  asymmetry of these modes is that the contribution which originates from the dipole term is independent of the Dalitz variables. Thus the key advantage of this class of final states over other probes of polarization observables in  $b \rightarrow q\gamma$  is that in this case the interpretation is relatively clean. In the case of  $b \rightarrow s\gamma$ , the SM contribution to  $CP$  violating observables is only a few percent, so a large signal would be an unmistakable sign of new physics. For  $b \rightarrow d\gamma$  the SM predicts a much smaller time dependent  $CP$  asymmetry so that this case may be viewed as a powerful null test.

In section II, we recall some of the basic issues in radiative decays. Corrections to the dominant dipole  $H_{\text{eff}}$  and their signals are also discussed. Section III briefly reviews the work of [6], whose generalization is the main focus of this paper. Sections IV and V contain the main body of our discussions on three-body modes. Section VI briefly mentions some generalizations and also presents experimental considerations. Section VII discusses the helicity of photons from the annihilation contribution; section VIII briefly discusses effect of non-dipole operators and section IX contains a brief summary. The possible complication in the analysis due to the presence of a perturbative phase is very briefly outlined in a short Appendix.

## II. BASICS OF RADIATIVE DECAYS

Let us consider radiative decays of the form  $B \rightarrow F\gamma$  where  $F$  is either a single meson (*e.g.*  $K^*$ ) or a multi-particle state (*e.g.*  $n\pi K$ ). The decay is governed by two amplitudes: the decay to right and left polarized photons; the same is true for the corresponding decay of the  $\bar{B}$ . We can denote these helicity amplitudes as follows:

$$\begin{aligned}
\overline{\mathcal{M}}_L &= \mathcal{M}(\overline{B} \rightarrow \overline{F}\gamma_L), \\
\overline{\mathcal{M}}_R &= \mathcal{M}(\overline{B} \rightarrow \overline{F}\gamma_R), \\
\mathcal{M}_R &= \mathcal{M}(B \rightarrow F\gamma_R), \\
\mathcal{M}_L &= \mathcal{M}(B \rightarrow F\gamma_L).
\end{aligned} \tag{1}$$

Here we adopt a phase convention for  $\gamma_{L,R}$  such that their phases are equal if the parity of the final state is opposite to the internal parity of  $F$ .

At short distances, the photons arise from a radiative transition of the  $b$  quark. In most models for this process arising from the electroweak scale or higher, it is to be expected that the dominant contribution to the  $b$ -scale effective Hamiltonian is via the dimension five dipole transition operator, which we will initially assume dominates the process:

$$H_{\text{eff}} = -\sqrt{8}G_F \frac{em_b}{16\pi^2} F_{\mu\nu} \left[ F_L^q \bar{q}\sigma^{\mu\nu} \frac{1+\gamma_5}{2} b + F_R^q \bar{q}\sigma^{\mu\nu} \frac{1-\gamma_5}{2} b \right] + h.c. \tag{2}$$

Here  $F_L^q$  ( $F_R^q$ ) corresponds to the amplitude for the emission of left (right) handed photons in the  $b_R \rightarrow q_L\gamma_L$  ( $b_L \rightarrow q_R\gamma_R$ ) decay, *i.e.* in the  $\overline{B} \rightarrow \overline{F}\gamma_L$  ( $\overline{B} \rightarrow \overline{F}\gamma_R$ ) decay. We can relate  $F_L^q$  and  $F_R^q$  by defining a parameter  $\psi^q$ , which is  $\mathcal{O}\left(\frac{m_q}{m_b}\right)$  in the SM:

$$\begin{aligned}
F_L^q &= F^q e^{i\phi_L^q} \cos\psi^q, \\
F_R^q &= F^q e^{i\phi_R^q} \sin\psi^q,
\end{aligned} \tag{3}$$

where  $\phi_L^q$  and  $\phi_R^q$  are  $CP$  violating phases. Since the strong interaction respects parity and charge conjugation, this model implies that the amplitudes are given in terms of a single complex valued form factor  $f$ , so that:

$$\begin{aligned}
\overline{\mathcal{M}}_L &= -F_L^q f(P\Phi_F)[P_{\text{internal}}], \\
\overline{\mathcal{M}}_R &= F_R^q f(\Phi_F), \\
\mathcal{M}_R &= F_L^{q*} f(C\Phi_F)[C_{\text{internal}}], \\
\mathcal{M}_L &= -F_R^{q*} f(CP\Phi_F)[(CP)_{\text{internal}}].
\end{aligned} \tag{4}$$

Here  $\Phi_F$  represents the phase space of the final state  $F$ . The sign in front of  $F_L^q$  and  $F_R^{q*}$  arises from the negative parity of the initial  $B^0$  or  $\overline{B}^0$  state. Also,  $P_{\text{internal}}$ ,  $C_{\text{internal}}$ ,  $(CP)_{\text{internal}}$  are the internal  $P$ ,  $C$ ,  $CP$  eigenvalues of all final state particles which are  $P$ ,  $C$ ,  $CP$  eigenstates. For example, in case of  $B_d \rightarrow K_S\pi^0\gamma_L$  and  $B_d \rightarrow K_S\pi^0\gamma_R$  decays, we obtain

$$\begin{aligned}
P_{\text{internal}} &= P_{K_S} P_{\pi^0} = (-1)(-1) = +1, \\
C_{\text{internal}} &= C_{K_S} C_{\pi^0} C_\gamma = (-1)(+1)(-1) = +1,
\end{aligned} \tag{5}$$

where we use  $J^{PC} = 0^{--}$  for  $K_S$  ignoring the tiny  $CP$  violation effect in the neutral kaon system. The strong phases in  $f$  arise from the rescattering between mesons in the final state  $F$ .

In the SM, the contributions are predominantly given by penguin diagrams such as that shown in Fig. 1. By CKM unitarity the short distance contribution to this has a CKM phase given by the phase of  $V_{tb}V_{ts}^*$ . This short distance contribution yields predominantly left-handed photons with the right handed component suppressed by  $m_s/m_b$ . This right-handed component will also have the same weak phase as the left-handed component.

The long distance (LD) contributions arise from the  $c$  and  $u$  penguins. They can have a nontrivial rescattering phase distinct from those present in the form factor  $f$  mentioned above. An example of a quark diagram contributing is shown in Fig. 2, which can also be understood as a rescattering of mesons through processes such as that shown in Fig. 3. Fig. 4 represents another LD contribution to radiative decays arising from “annihilation” diagrams; we shall discuss this particular contribution later in the paper.

The precise effect of such LD processes is difficult to calculate reliably. In [18] a detailed estimate in the case of  $B \rightarrow V\gamma$  is given and it is found that the photons from these contributions are still predominantly left-handed. On the other hand, the authors of [15] entertain the possibility that the contribution is of the form  $\mathcal{M}_L = -\mathcal{M}_R$ .

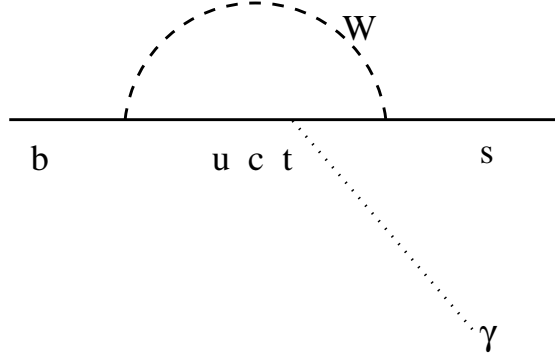


FIG. 1: A typical SM radiative penguin graph.

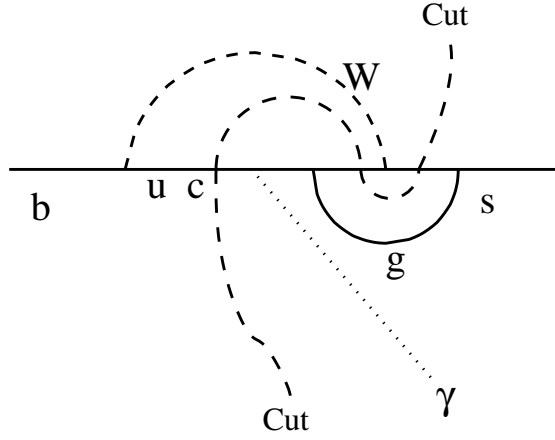


FIG. 2: An example of a QCD loop correction which generates the absorptive part necessary for direct  $CP$  asymmetry, see [5]. The cut is indicated by the dashed line.

In the case of  $b \rightarrow s\gamma$  such a contribution will be suppressed with respect to the short distance [21]. However, for  $b \rightarrow d\gamma$  the magnitude of such a contribution may be quite appreciable. Phenomenologically, the primary manifestation of such “long distance” contributions would be partial rate asymmetries in  $B \rightarrow K^*\gamma$  or  $B \rightarrow \rho\gamma$ .

In such cases when there is a contribution which has a strong phase, it can result in a contribution to Eqn. 4 which although cannot readily be reliably calculated, may be parameterized as follows:

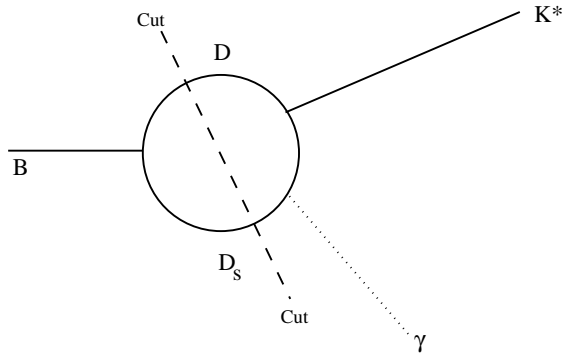


FIG. 3: A possible long distance rescattering effect due to on-shell  $D_s\overline{D}$  contribution, see reference [18].

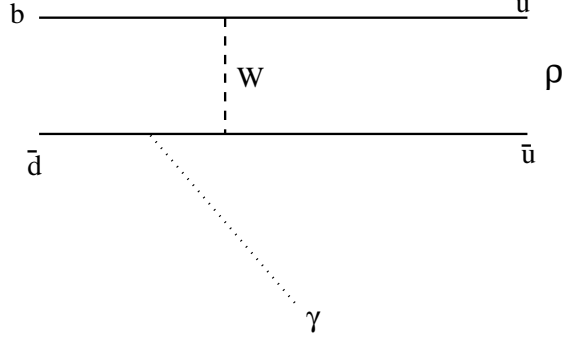


FIG. 4: The dominant contribution from the annihilation graph, see reference [19].

$$\begin{aligned}
\overline{\mathcal{M}}_L &= -F_L^q f(P\Phi_F)(1 + \overline{\delta}_L(P\Phi_F))[P_{\text{internal}}], \\
\overline{\mathcal{M}}_R &= F_R^q f(\Phi_F)(1 + \overline{\delta}_R(\Phi_F)), \\
\mathcal{M}_R &= F_L^{q*} f(C\Phi_F)(1 + \delta_R(C\Phi_F))[C_{\text{internal}}], \\
\mathcal{M}_L &= -F_R^{q*} f(CP\Phi_F)(1 + \delta_L(CP\Phi_F))[(CP)_{\text{internal}}],
\end{aligned} \tag{6}$$

where the  $\delta$  terms are arbitrary complex form factors. If such contributions are only from the SM and the photon is predominantly the same as the short distance SM effects then:

$$\begin{aligned}
\delta_L &= \overline{\delta}_R = 0, \\
\delta_R &= e^{i\mu}\Delta, \\
\overline{\delta}_L &= e^{-i\mu}\Delta,
\end{aligned} \tag{7}$$

where  $\mu$  is the weak phase between the charm and top penguins and  $\Delta$  is a complex valued function of  $\Phi_F$ . For  $b \rightarrow s\gamma$  the small direct partial rate asymmetry from the SM may also be eventually detectable [5, 17]. In the case of  $b \rightarrow d\gamma$  the direct  $CP$  asymmetry can be quite sizable.

In general, new physics should assert itself as an additional contribution to  $F^q$  and  $\phi$  in Eqn. 3 which is different from the SM and we will assume therefore that new physics is only manifest at short distances. If this were not the case, then probably other signals would be more suited for its detection. Since our methods are generalizations of those proposed in AGS [6], let us now briefly review that method.

### III. AGS OSCILLATION

Before proceeding it is useful to consider the conditions under which the AGS oscillations occur. These follow from the general conditions for oscillating signals in neutral mesons applied specifically to the case of radiative decays. Thus for there to be oscillations in the decay  $B \rightarrow V\gamma$  the following conditions are necessary:

- (1) Both  $B$  and  $\overline{B}$  must decay to the same exclusive final state  $V\gamma$  (*e.g.*  $V = K^*, \rho$  or higher resonances).
- (2) The photons produced in  $B \rightarrow V\gamma$  or  $\overline{B} \rightarrow V\gamma$  must be a mixture of right and left handed helicities.

Assuming these conditions are met, we recall that in general we can describe the time dependent wave function of the  $B$  meson (either  $B_d$  or  $B_s$ ) as [22]:

$$\begin{aligned}
|B(t)\rangle &= g_+|B\rangle + \frac{q}{p}g_-|\overline{B}\rangle, \\
|\overline{B}(t)\rangle &= g_+|\overline{B}\rangle + \frac{p}{q}g_-|B\rangle,
\end{aligned} \tag{8}$$

where

$$g_{\pm} = \frac{1}{2} e^{-iM_1 t} e^{-\frac{1}{2}\Gamma_1 t} \left[ 1 \pm e^{-i\Delta m t} e^{\frac{1}{2}\Delta\Gamma t} \right]. \quad (9)$$

In the case of  $B_d$  decay, we assume that  $q/p = e^{i\phi_M}$ , with  $\phi_M = -2\phi_1$  [24] in the SM, and that  $\Delta\Gamma_d$  is small. In this limit we can write the time dependent decay rates of  $B_d$  and  $\bar{B}_d$  to given final state  $V\gamma$  as:

$$\begin{aligned} \Gamma_{B_d \rightarrow V\gamma}(t) &\equiv \Gamma_{B_d \rightarrow V\gamma_L}(t) + \Gamma_{B_d \rightarrow V\gamma_R}(t) \propto e^{-\Gamma t} [X_{V\gamma} + Y_{V\gamma} \cos(\Delta m_d t) + Z_{V\gamma} \sin(\Delta m_d t)], \\ \Gamma_{\bar{B}_d \rightarrow V\gamma}(t) &\equiv \Gamma_{\bar{B}_d \rightarrow V\gamma_L}(t) + \Gamma_{\bar{B}_d \rightarrow V\gamma_R}(t) \propto e^{-\Gamma t} [X_{V\gamma} - Y_{V\gamma} \cos(\Delta m_d t) - Z_{V\gamma} \sin(\Delta m_d t)], \end{aligned} \quad (10)$$

where

$$\begin{aligned} X_{V\gamma} &= (|\mathcal{M}_L|^2 + |\mathcal{M}_R|^2) + (|\overline{\mathcal{M}}_L|^2 + |\overline{\mathcal{M}}_R|^2), \\ Y_{V\gamma} &= (|\mathcal{M}_L|^2 + |\mathcal{M}_R|^2) - (|\overline{\mathcal{M}}_L|^2 + |\overline{\mathcal{M}}_R|^2), \\ Z_{V\gamma} &= -2 \text{Im} (e^{i\phi_M} (\mathcal{M}_L^* \overline{\mathcal{M}}_L + \mathcal{M}_R^* \overline{\mathcal{M}}_R)). \end{aligned} \quad (11)$$

Here we sum decay rates for the left-handed and right-handed photon helicity states as we do not distinguish between the two. We also define  $CP$  violation parameters  $\mathcal{S}_{V\gamma}$  and  $\mathcal{A}_{V\gamma}$  as [25]

$$\frac{\Gamma_{\bar{B}_d \rightarrow V\gamma}(t) - \Gamma_{B_d \rightarrow V\gamma}(t)}{\Gamma_{\bar{B}_d \rightarrow V\gamma}(t) + \Gamma_{B_d \rightarrow V\gamma}(t)} \equiv \mathcal{S}_{V\gamma} \sin(\Delta m_d t) + \mathcal{A}_{V\gamma} \cos(\Delta m_d t). \quad (12)$$

The parameter  $\mathcal{S}_{V\gamma}$  represents mixing-induced  $CP$  violation, while  $\mathcal{A}_{V\gamma}$  represents direct  $CP$  violation. From Eqn. 10, we obtain  $\mathcal{S}_{V\gamma} = -Z_{V\gamma}/X_{V\gamma}$  and  $\mathcal{A}_{V\gamma} = -Y_{V\gamma}/X_{V\gamma}$ .

In AGS it was assumed that  $V\gamma$  was a vector-photon ( $K^*\gamma$  or  $\rho\gamma$ ) final state and that the photon emission was described by the short distance Hamiltonian of Eqn. 2. Note that to respect the conditions for oscillations, the  $K^*$  state must be in a  $C$  eigenstate which is only true if  $K^* \rightarrow K_S \pi^0$  or  $K^* \rightarrow K_L \pi^0$  (the latter is unlikely to be experimentally useful). Assuming that the polarization information is not available, we must content ourselves with summing over polarization and, neglecting long distance contributions, thus obtain the time dependent forms:

$$\begin{aligned} \Gamma_{B_d \rightarrow V\gamma}(t) &\propto e^{-\Gamma t} [1 + \chi_V \sin 2\psi^q \sin \phi^q \sin(\Delta m_d t)], \\ \Gamma_{\bar{B}_d \rightarrow V\gamma}(t) &\propto e^{-\Gamma t} [1 - \chi_V \sin 2\psi^q \sin \phi^q \sin(\Delta m_d t)], \end{aligned} \quad (13)$$

where  $\phi^q = \phi_M + \phi_L + \phi_R$  [26],  $\chi_V$  denotes the  $C$  eigenvalue of  $V$  [27] and the superscript  $q$  indicates the quark produced in the  $b \rightarrow q\gamma$  decay.

This oscillation therefore allows the extraction of the quantity  $\mathcal{S}_{V\gamma} = -\chi_V \sin 2\psi^q \sin \phi^q$ . In the case of  $B_d \rightarrow K^*\gamma$  where the short distance contribution to the photon is predominantly right handed,  $\psi^s$  is small and so  $\mathcal{S}_{K^*\gamma}$  is consequently small. The observation of a significant  $\mathcal{S}_{K^*\gamma}$  would therefore indicate the presence of NP. To the extent that the long distance contribution to the photon is right handed as suggested by the calculation of [18], then the same is also true of  $B_d \rightarrow \rho\gamma$ .

Let us now consider the generalization to final states with two pseudoscalars, in particular  $K_S \pi^0 \gamma$  and  $\pi^+ \pi^- \gamma$ . Note that the case of  $K^*\gamma$  with  $K^* \rightarrow K_S \pi^0$  is a special case of  $K_S \pi^0 \gamma$  where the two mesons are on the  $K^*$  resonance while  $\rho^0 \gamma$  is a special case of  $\pi^+ \pi^- \gamma$  with the two mesons on the  $\rho$  resonance. For clarity, let us remark that any kaonic resonance of angular momentum  $J$ , that produces  $K_S \pi^0$ , will have  $P = (-1)^J$  with  $C = -1$ ; thus for any such resonance  $\chi_V = -1$  and so the oscillations for all of them will be identical as they are all governed by Eqn. 13.

In contrast, for a  $\pi^+ \pi^-$  resonance of spin  $J$ ,  $P = (-1)^J$  and (since  $CP = +$ )  $C = (-1)^J$ . It follows then that  $\chi_V = (-1)^J$ , so all odd- $J$  and even- $J$  resonances will have opposite signs in Eqn. 13.

In the next section we will generalize these results to be independent of the resonance the mesons might go through and consider the angular distribution of the pseudoscalars.

#### IV. $B_d \rightarrow K_S \pi^0 \gamma$ AND $B_d \rightarrow \pi^+ \pi^- \gamma$

In this section we will contrast the nature of AGS oscillations in the cases of  $B_d \rightarrow K_S \pi^0 \gamma$  [28] and  $B_d \rightarrow \pi^+ \pi^- \gamma$ . The discussion here is easily generalized to the case of other (pseudo-)scalar pairs recoiling against the photon. We will consider the more general case including direct  $CP$  violation in the next section.

The key point to realize is that there is a contrast between the symmetry properties of phase space in the case of  $K_S\pi^0$  with that from  $\pi^+\pi^-$ . To see how this arises let us designate the  $K_S$  to be particle “1” and the  $\pi^0$  to be particle “2” in the first case while we designate  $\pi^+$  as particle “1” and  $\pi^-$  as particle “2” in the second case. For  $K_S\pi^0\gamma$  then:

$$\begin{aligned} C\Phi(\gamma_L^R, 1, 2) &= \Phi(\gamma_L^R, 1, 2), \\ P\Phi(\gamma_L^R, 1, 2) &= \Phi(\gamma_R^L, 1, 2), \end{aligned} \quad (14)$$

while for  $\pi^+\pi^-\gamma$ ,

$$\begin{aligned} C\Phi(\gamma_L^R, 1, 2) &= \Phi(\gamma_L^R, 2, 1), \\ P\Phi(\gamma_L^R, 1, 2) &= \Phi(\gamma_R^L, 1, 2). \end{aligned} \quad (15)$$

That is to say, under the  $C$  and  $P$  transformations, each point in the  $K_S\pi^0\gamma$  phase space is translated into the same point (notwithstanding the distinct photon helicity under  $P$  and  $CP$ ), whereas for  $\pi^+\pi^-\gamma$  the positions of the pions are interchanged under  $C$  and  $CP$ .

It is now a simple matter to generalize the AGS formalism to the case where the final amplitude is also a function of phase space. Consider first the case of  $B_d \rightarrow K_S\pi^0\gamma$ , which at the quark level corresponds to  $\bar{b} \rightarrow \bar{s}\gamma$ . The decay amplitude consists of two components which do not interfere with each other corresponding to photons with left and right handed helicities. These amplitudes should depend on the Dalitz plot variables which we will denote:

$$\begin{aligned} s_1 &= (p_{K_S} + p_{\pi_0})^2, \\ s_2 &= (p_{K_S} + p_\gamma)^2, \\ s_3 &= (p_\gamma + p_{\pi_0})^2, \\ z &= \frac{s_3 - s_2}{s_3 + s_2}. \end{aligned} \quad (16)$$

In particular the amplitude can be expressed as a function of  $s_1$  and  $z$ , where  $s_1$  is the invariant mass squared of the  $K_S\pi^0$  system, and  $z$  is the cosine of the angle between the  $B_d$  and  $\pi^0$  in the  $K_S\pi^0$  frame.

If we assume that the decay  $B_d \rightarrow K_S\pi^0\gamma$  is governed by Eqn. 2 then the amplitude as a function of  $s_1$  and  $z$  can be written from Eqn. 4 as [29]:

$$\begin{aligned} \mathcal{M}_R(s_1, z) &= F_L^{s*} f_R(s_1, z), \\ \mathcal{M}_L(s_1, z) &= F_R^{s*} f_L(s_1, z), \end{aligned} \quad (17)$$

where the subscript on  $\mathcal{M}$  indicates the helicity of the photon emitted and the superscript indicates the quark produced by the reaction (*i.e.*  $d$  or  $s$ ). For the charge conjugate decay  $\bar{B}_d \rightarrow K_S\pi^0\gamma$  we can likewise write:

$$\begin{aligned} \overline{\mathcal{M}}_R(s_1, z) &= F_R^s \bar{f}_R(s_1, z), \\ \overline{\mathcal{M}}_L(s_1, z) &= F_L^s \bar{f}_L(s_1, z). \end{aligned} \quad (18)$$

Since QCD respects both  $C$  and  $P$ , we expect from Eqn. 4

$$f_R(s_1, z) = \bar{f}_R(s_1, z) = -f_L(s_1, z) = -\bar{f}_L(s_1, z). \quad (19)$$

Thus we obtain

$$\begin{aligned} \overline{\mathcal{M}}_L(s_1, z) &= -F_L^s f(s_1, z), \\ \overline{\mathcal{M}}_R(s_1, z) &= F_R^s f(s_1, z), \\ \mathcal{M}_R(s_1, z) &= F_L^{s*} f(s_1, z), \\ \mathcal{M}_L(s_1, z) &= -F_R^{s*} f(s_1, z), \end{aligned} \quad (20)$$

where we define a universal form factor  $f(s_1, z) = f_R(s_1, z)$ . Note that in this discussion of the amplitudes at a fixed point  $(s_1, z)$  in phase space, the relative angular momentum between  $K_S$  and  $\pi^0$  does not enter and therefore the quantum numbers (*e.g.* spin) of intermediate kaonic resonances contributing to the process do not effect the conclusions [30]. This is somewhat different from the behavior, discussed below, for the case where the two final state mesons are antiparticles *e.g.*  $B_d \rightarrow \pi^+ \pi^- \gamma$ .

The time dependent rates for physical  $B_d$  and  $\bar{B}_d$  decays, at a point in phase space defined by  $(s_1, z)$ , and summed over photon helicity are therefore given by:

$$\begin{aligned} \Gamma_{B_d \rightarrow K_S \pi^0 \gamma}(t, s_1, z) &\propto e^{-\Gamma t} [X_{K_S \pi^0 \gamma}(s_1, z) + Y_{K_S \pi^0 \gamma}(s_1, z) \cos(\Delta m_d t) + Z_{K_S \pi^0 \gamma}(s_1, z) \sin(\Delta m_d t)], \\ \Gamma_{\bar{B}_d \rightarrow K_S \pi^0 \gamma}(t, s_1, z) &\propto e^{-\Gamma t} [X_{K_S \pi^0 \gamma}(s_1, z) - Y_{K_S \pi^0 \gamma}(s_1, z) \cos(\Delta m_d t) - Z_{K_S \pi^0 \gamma}(s_1, z) \sin(\Delta m_d t)], \end{aligned} \quad (21)$$

where, neglecting long distance effects,

$$\begin{aligned} X_{K_S \pi^0 \gamma}(s_1, z) &= 2(F^s)^2 |f(s_1, z)|^2, \\ Y_{K_S \pi^0 \gamma}(s_1, z) &= 0, \\ Z_{K_S \pi^0 \gamma}(s_1, z) &= -2(F^s)^2 |f(s_1, z)|^2 \sin 2\psi^s \sin \phi^s, \end{aligned} \quad (22)$$

and  $\phi^s = \phi_M + \phi_L^s + \phi_R^s$  is the weak phase.

Thus, for each value of  $s_1$  and  $z$ , the  $CP$  asymmetry is:

$$\frac{\Gamma_{\bar{B}_d \rightarrow K_S \pi^0 \gamma} - \Gamma_{B_d \rightarrow K_S \pi^0 \gamma}}{\Gamma_{\bar{B}_d \rightarrow K_S \pi^0 \gamma} + \Gamma_{B_d \rightarrow K_S \pi^0 \gamma}} = +\sin 2\psi^s \sin \phi^s \sin(\Delta m_d t). \quad (23)$$

Note that this expression is true whether the  $K_S \pi^0$  is produced by the decay of a resonance or is nonresonant. Furthermore, the fact that the effective Hamiltonian of Eqn. 2 produces the photons implies that this asymmetry does not depend on  $s_1$ . In effect each point in phase space is a separate oscillation experiment which shows the same oscillator behavior given by Eqn. 23. In practice this means that one may add together all events of the form  $B_d \rightarrow K_S \pi^0 \gamma$  regardless of whether they are produced at the  $K^*(892)$  resonance, some other resonance (*e.g.*  $K_2^*(1430)$ ) or from a nonresonant source and determine the single quantity  $\mathcal{S}_{B_d \rightarrow K_S \pi^0 \gamma} = +\sin 2\psi^s \sin \phi^s$  as a result. Within the SM, we obtain  $\mathcal{S}_{B_d \rightarrow K_S \pi^0 \gamma} \approx -(2m_s/m_b) \sin 2\phi_1$  as  $\psi \approx m_s/m_b$  and  $\phi^s = -2\phi_1 + O(\lambda^2)$ , are expected where  $\lambda \approx 0.22$  is the sine of the Cabibbo angle. In terms of the individual parameters this gives the lower bounds:

$$\begin{aligned} |\sin 2\psi^s| &\geq |\mathcal{S}_{B_d \rightarrow K_S \pi^0 \gamma}|, \\ |\sin \phi^s| &\geq |\mathcal{S}_{B_d \rightarrow K_S \pi^0 \gamma}|. \end{aligned} \quad (24)$$

Deviations from this picture of uniform oscillation as a function of phase space would indicate contributions to the radiative decay other than the pure dipole transition of the effective Hamiltonian of Eqn. 2.

Consider now the case of  $B_d \rightarrow \pi^+ \pi^- \gamma$ . Again for the  $B_d$  decays we can define the Dalitz variables:

$$\begin{aligned} s_1 &= (p_{\pi^+} + p_{\pi^-})^2, \\ s_2 &= (p_{\pi^+} + p_\gamma)^2, \\ s_3 &= (p_\gamma + p_{\pi^-})^2, \\ z &= \frac{s_3 - s_2}{s_3 + s_2}, \end{aligned} \quad (25)$$

and thus write the amplitudes in the form:

$$\begin{aligned} \overline{\mathcal{M}}_L(s_1, z) &= F_L^d \overline{g}_L(s_1, z), \\ \overline{\mathcal{M}}_R(s_1, z) &= F_R^d \overline{g}_R(s_1, z), \\ \mathcal{M}_R(s_1, z) &= F_L^{d*} g_R(s_1, z), \\ \mathcal{M}_L(s_1, z) &= F_R^{d*} g_L(s_1, z). \end{aligned} \quad (26)$$



As before  $g_L = -g_R$  but in this case in applying charge conjugation to get from  $g_L$  to  $\bar{g}_L$  we need to interchange the coordinates of the  $\pi^+$  and  $\pi^-$ . Thus  $s_2 \leftrightarrow s_3$  so  $z \leftrightarrow -z$ , therefore [31]:

$$\begin{aligned} g_L(s_1, z) &= -g_R(s_1, z) \equiv -g(s_1, -z), \\ \bar{g}_L(s_1, z) &= -\bar{g}_R(s_1, z) = -g(s_1, z). \end{aligned} \quad (27)$$

For particular partial waves of the  $\pi^+\pi^-$  system with angular momentum  $L$ ,  $g(s_1, z) = (-1)^{L+1}g(s_1, -z)$ . In general  $g$  will be a mixture of even and odd  $L$ , so  $g$  has no particular symmetry under  $z \leftrightarrow -z$ . Note also that  $g$  will in general have a nontrivial  $CP$ -even phase which depends on  $s_1$  and  $z$ . The time dependent rates at a point in phase space defined by  $(s_1, z)$ , and summed over helicity will be given by:

$$\begin{aligned} \Gamma_{B_d \rightarrow \pi^+\pi^-\gamma}(t, s_1, z) &\propto e^{-\Gamma t} [X_{\pi^+\pi^-\gamma}(s_1, z) + Y_{\pi^+\pi^-\gamma}(s_1, z) \cos(\Delta m_d t) + Z_{\pi^+\pi^-\gamma}(s_1, z) \sin(\Delta m_d t)], \\ \Gamma_{\bar{B}_d \rightarrow \pi^+\pi^-\gamma}(t, s_1, z) &\propto e^{-\Gamma t} [X_{\pi^+\pi^-\gamma}(s_1, z) - Y_{\pi^+\pi^-\gamma}(s_1, z) \cos(\Delta m_d t) - Z_{\pi^+\pi^-\gamma}(s_1, z) \sin(\Delta m_d t)]. \end{aligned} \quad (28)$$

Here:

$$\begin{aligned} X_{\pi^+\pi^-\gamma}(s_1, z) &= (F^d)^2 [|g(s_1, z)|^2 + |g(s_1, -z)|^2], \\ Y_{\pi^+\pi^-\gamma}(s_1, z) &= (F^d)^2 [|g(s_1, z)|^2 - |g(s_1, -z)|^2], \\ Z_{\pi^+\pi^-\gamma}(s_1, z) &= -2(F^d)^2 [\text{Re}(g(s_1, -z)g^*(s_1, z)) \sin \phi^d + \text{Im}(g(s_1, -z)g^*(s_1, z)) \cos \phi^d] \sin 2\psi^d, \end{aligned} \quad (29)$$

and  $\phi^d = \phi_M + \phi_L^d + \phi_R^d$  is the weak phase; in the SM  $\phi_M \approx -2\phi_1$  and  $\phi_L^d = \phi_R^d \sim \phi_1$ .

At each point in phase space,  $g(s_1, z)$  and  $g(s_1, -z)$  are, in general, independent complex numbers. Unlike the case of  $K_S\pi^0\gamma$ , this case does, however, allow the possibility of extracting  $\phi^d$  and  $\psi^d$  separately up to discrete ambiguities. This can be achieved without any assumption about the resonant structure of the  $\pi^+\pi^-\gamma$  final state, as we now demonstrate. Note that our argument requires regions of phase space where  $\pi^+\pi^-$  partial waves with different angular momentum interfere, otherwise  $g(s_1, z) = \pm g(s_1, -z)$  everywhere and consequently there is no additional information compared to the  $K_S\pi^0\gamma$  case.

For a given value of  $s_1$  and  $z$  let us define:

$$\begin{aligned} u &= \cos 2\psi^d, & a &= \frac{|g(s_1, z)|^2 - |g(s_1, -z)|^2}{|g(s_1, z)|^2 + |g(s_1, -z)|^2}, \\ v &= \sin 2\psi^d \sin \phi^d, & b &= -2 \frac{\text{Re}(g^*(s_1, z)g(s_1, -z))}{|g(s_1, z)|^2 + |g(s_1, -z)|^2}, \\ w &= \sin 2\psi^d \cos \phi^d, & c &= -2 \frac{\text{Im}(g^*(s_1, z)g(s_1, -z))}{|g(s_1, z)|^2 + |g(s_1, -z)|^2}, \end{aligned} \quad (30)$$

where  $a^2 + b^2 + c^2 = u^2 + v^2 + w^2 = 1$ . Recalling that the experimental observables,  $X_{\pi^+\pi^-\gamma}$ ,  $Y_{\pi^+\pi^-\gamma}$  and  $Z_{\pi^+\pi^-\gamma}$  are functions of the phase space, we can relate these via:

$$\begin{aligned} \eta &= \frac{Y_{\pi^+\pi^-\gamma}}{X_{\pi^+\pi^-\gamma}} = a, \\ \zeta_+ &= \frac{Z_{\pi^+\pi^-\gamma} + Z_{\pi^+\pi^-\gamma}}{2X_{\pi^+\pi^-\gamma}} = bv, \\ \zeta_- &= \frac{Z_{\pi^+\pi^-\gamma} - Z_{\pi^+\pi^-\gamma}}{2X_{\pi^+\pi^-\gamma}} = cw. \end{aligned} \quad (31)$$

Along the line  $z = 0$ , we find  $\zeta_- = \eta = 0$  while  $\zeta_+ = -v$ . Once  $v$  is known, we can learn  $w$  from cases where  $z \neq 0$ :

$$w = \frac{\pm \zeta_-}{\sqrt{1 - \left(\frac{\zeta_+}{v}\right)^2 - \eta^2}} = \frac{\pm \zeta_-}{\sqrt{1 - \left(\frac{\zeta_+}{\zeta_+(z=0)}\right)^2 - \eta^2}}. \quad (32)$$

From here we can determine  $\phi^d$  and  $\psi^d$  through

$$\begin{aligned} \sin^2 2\psi^d &= v^2 + w^2, \\ \phi^d &= \arg((w + iv)/\sin(2\psi^d)). \end{aligned} \quad (33)$$

There are 8 solutions for  $\psi^d$ . Each of them has two  $\phi^d$  values corresponding to positive and negative solutions for  $w$ . Thus there are 16 valid  $(\psi^d, \phi^d)$  combinations in total.

In the preceding discussion, we have left the functions  $g_{L,R}$  arbitrary. In practice, the  $\pi^+\pi^-$  amplitude is very likely to be dominated by low-lying resonances with well known masses, widths and quantum numbers. This knowledge could facilitate a more constrained fit to the data. In case  $B_d \rightarrow \rho^0(\rightarrow \pi^+\pi^-)\gamma$  is dominant, we expect  $g(s_1, z) = g(s_1, -z)$  as the  $\pi^+\pi^-$  system is in the  $L$ -odd state. Substituting this into Eqn. 29, we obtain  $\mathcal{S}_{B_d \rightarrow \rho^0 \gamma} = +\sin 2\psi^d \sin 2\phi^d$ .

Throughout this discussion we have, again, assumed dominance of the dipole operators in  $H_{\text{eff}}$ , as in Eqn. 2. To the extent that this assumption holds,  $\psi$  and  $\phi$  would be independent of the Dalitz variables,  $s_1$  and  $z$ , as before. Conversely, variation of  $\psi$  and  $\phi$  with  $s_1$  and  $z$  would shed light on the contribution from non-dipole interactions.

### V. $B_s \rightarrow K^+K^-\gamma$ AND $B_s \rightarrow K_S\pi^0(\eta^{(\prime)})\gamma$

Let us consider now the analogous  $B_s$  decays. In this case we cannot assume that  $\Delta\Gamma \approx 0$ ; then the time dependent decay rates to a final state  $F\gamma$  are given by:

$$\begin{aligned} \Gamma_{B_s \rightarrow F\gamma}(t) &\propto e^{-\Gamma t} \left[ X_{F\gamma} \cosh(\tfrac{1}{2}\Delta\Gamma_s t) + Y_{F\gamma} \cos(\Delta m_s t) + \right. \\ &\quad \left. Z_{F\gamma} \sin(\Delta m_s t) + W_{F\gamma} \sinh(\tfrac{1}{2}\Delta\Gamma_s t) \right], \\ \Gamma_{\bar{B}_s \rightarrow F\gamma}(t) &\propto e^{-\Gamma t} \left[ X_{F\gamma} \cosh(\tfrac{1}{2}\Delta\Gamma_s t) - Y_{F\gamma} \cos(\Delta m_s t) - \right. \\ &\quad \left. Z_{F\gamma} \sin(\Delta m_s t) + W_{F\gamma} \sinh(\tfrac{1}{2}\Delta\Gamma_s t) \right], \end{aligned} \quad (34)$$

where

$$\begin{aligned} X_{F\gamma} &= (|\mathcal{M}_L|^2 + |\mathcal{M}_R|^2) + (|\overline{\mathcal{M}}_L|^2 + |\overline{\mathcal{M}}_R|^2), \\ Y_{F\gamma} &= (|\mathcal{M}_L|^2 + |\mathcal{M}_R|^2) - (|\overline{\mathcal{M}}_L|^2 + |\overline{\mathcal{M}}_R|^2), \\ Z_{F\gamma} &= -2 \text{Im} (e^{i\phi_M} (\mathcal{M}_L^* \overline{\mathcal{M}}_L + \mathcal{M}_R^* \overline{\mathcal{M}}_R)), \\ W_{F\gamma} &= -2 \text{Re} (e^{i\phi_M} (\mathcal{M}_L^* \overline{\mathcal{M}}_L + \mathcal{M}_R^* \overline{\mathcal{M}}_R)). \end{aligned} \quad (35)$$

Here  $\phi_M \approx O(\lambda^2)$  is expected for the  $B_s$ - $\bar{B}_s$  mixing in the SM (in the usual phase convention).

Let us first consider the decay to two mesons which are self-conjugate such as  $B_s \rightarrow K_S\pi^0\gamma$ . The expression Eqn. 34 also describes the time dependent decay rates for physical  $B_s$  and  $\bar{B}_s$  decays at a point in phase space defined by  $(s_1, z)$ , and summed over photon helicity. Neglecting long distance effects, the factors  $X$ ,  $Y$ ,  $Z$  and  $W$  are given by

$$\begin{aligned} X_{K_S\pi^0}(s_1, z) &= 2(F^d)^2 |h(s_1, z)|^2, \\ Y_{K_S\pi^0}(s_1, z) &= 0, \\ Z_{K_S\pi^0}(s_1, z) &= -2(F^d)^2 |h(s_1, z)|^2 \sin 2\psi^d \sin \phi^d, \\ W_{K_S\pi^0}(s_1, z) &= -2(F^d)^2 |h(s_1, z)|^2 \sin 2\psi^d \cos \phi^d. \end{aligned} \quad (36)$$

In this case, if the value of  $W$  can be isolated, both angles  $\phi^d$  and  $\psi^d$  can be determined, up to discrete ambiguities. Of course if  $\Delta\Gamma/\Gamma$  is relatively small, this may prove difficult in practice. As in the case of  $B_d \rightarrow K_S\pi^0\gamma$ , the asymmetries are independent of  $s_1$  and  $z$ .

In the case of  $B_s \rightarrow K^+K^-\gamma$ , the phase space dependent oscillation is again given by the expressions in Eqn. 34. The  $X$ ,  $Y$ ,  $Z$  and  $W$  terms are given by:

$$\begin{aligned} X_{K^+K^-}(s_1, z) &= (F^s)^2 (|d(s_1, z)|^2 + |d(s_1, -z)|^2), \\ Y_{K^+K^-}(s_1, z) &= (F^s)^2 (|d(s_1, z)|^2 - |d(s_1, -z)|^2), \\ Z_{K^+K^-}(s_1, z) &= -2(F^s)^2 \{ \text{Re} (d(s_1, -z) d^*(s_1, z)) \sin \phi^s + \text{Im} (d(s_1, -z) d^*(s_1, z)) \cos \phi^s \} \sin 2\psi^s, \\ W_{K^+K^-}(s_1, z) &= -2(F^s)^2 \{ \text{Re} (d(s_1, -z) d^*(s_1, z)) \cos \phi^s - \text{Im} (d(s_1, -z) d^*(s_1, z)) \sin \phi^s \} \sin 2\psi^s. \end{aligned} \quad (37)$$

From the  $X$ ,  $Y$  and  $Z$  terms one can determine  $\phi^s$  and  $\psi^s$  as described above in the  $B_d \rightarrow \pi^+\pi^-\gamma$  case. In addition, if the  $W$  term can be determined it allows for the resolution of the ambiguity between  $\phi^s$ ,  $\psi^s$  and  $\phi^s$ ,  $-\psi^s$ .

	$K_S\pi^0\gamma$	$K_SK_S\gamma$	$\pi^+\pi^-\gamma$	$K^+K^-\gamma$	$K_SK_L\gamma$
$B_d/\overline{B}_d$	$b \rightarrow s\gamma$	$b \rightarrow d\gamma$	$b \rightarrow d\gamma$	$b \rightarrow d\gamma$	$b \rightarrow d\gamma$
$B_s/\overline{B}_s$	$b \rightarrow d\gamma$	$b \rightarrow s\gamma$	$b \rightarrow s\gamma$	$b \rightarrow s\gamma$	$b \rightarrow s\gamma$

TABLE I: Final states which can be used to probe  $b \rightarrow s\gamma$  and  $b \rightarrow d\gamma$  transitions in  $B_d$  and  $B_s$  decays. This list is not exhaustive; in particular other neutral (pseudo-)scalar particles ( $\eta$ ,  $\eta'$ ,  $f_0$ ) may be used in the place of  $\pi^0$ .

## VI. ANALOGOUS CASES, GENERALIZATIONS AND EXPERIMENTAL CONSIDERATIONS

In the above discussion we have considered some instances of  $B_d \rightarrow P_1 P_2 \gamma$  and  $B_s \rightarrow P_1 P_2 \gamma$  where  $P_i$  represent scalar or pseudoscalar states. The mode of analysis depends on the nature of  $P_1 P_2$ .

If there is no relation between  $P_1$  and  $P_2$  and they are not self-conjugate then there will be no oscillations. For instance in the case of  $B_d \rightarrow K^+ \pi^- \gamma$  we can tell from the final state whether the initial state is  $B$  or  $\overline{B}$  so no quantum interference is possible. Final states of this sort do however give the simplest way to determine if there is direct  $CP$  violation at the quark level.

If  $P_1$  and  $P_2$  are both eigenstates of charge conjugation, then the mode of analysis is the same as  $B_d \rightarrow K_S \pi^0 \gamma$ . In this case we can learn the product  $\sin 2\psi^q \sin \phi^q$ , where the superscript  $q$  represents the quark produced in  $b \rightarrow q\gamma$ . For  $B_s$  oscillations, if the  $W$  term can be extracted then we can learn  $\phi^q$  and  $\psi^q$  separately. In all cases, the data can be integrated over  $s_1$  and  $z$ . Some other final states of this type, relevant to  $B_d$  decays, are  $K_S \eta^{(\prime)} \gamma$ ,  $K_S f_0 \gamma$ ,  $\pi^0 \eta^{(\prime)} \gamma$ ,  $\pi^0 f_0 \gamma$ . For most of these modes there is currently no experimental information; they are therefore in urgent need of investigation. Cases that may be of special interest are  $B^0 \rightarrow K_S \eta' \gamma$  and  $B^0 \rightarrow K_S \eta \gamma$ . Comparisons to the pattern of branching fractions in two body hadronic  $B$  decay suggest that the nonresonant contribution to  $B \rightarrow K \eta' \gamma$  ought to be larger than that for either  $B \rightarrow K \eta \gamma$  or  $B \rightarrow K \pi^0 \gamma$ . Note also that  $B \rightarrow K \eta \gamma$  has recently been observed with branching ratio  $\approx 7 \times 10^{-6}$  [32]. Therefore it is possible that an appreciable data sample for  $K_S \eta' \gamma$  may already be available with the current  $B$  factory statistics.

Note that there is a set of special cases of these modes, where  $P_1 = P_2$ , for instance,  $K_S K_S \gamma$  and  $\pi^0 \pi^0 \gamma$  (the latter unlikely to be of practical use unless a very high luminosity allows us to use  $\pi^0 \rightarrow e^+ e^- \gamma$  decays that provide vertex information). In these cases, Bose-Einstein statistics further constrain the  $P_1 P_2$  system [9].

As explained in Sec. IV, the relative angular momentum between  $P_1$  and  $P_2$  does not enter; only the intrinsic charge conjugation quantum numbers of  $P_1$  and  $P_2$  affect  $CP$  asymmetries. This is also valid in more general cases with more than two particles, *e.g.*  $B \rightarrow P_1 P_2 P_3 \gamma$ , where  $P_1$ ,  $P_2$  and  $P_3$  are eigenstates of charge conjugation. Specific examples of this type are  $B \rightarrow K_S \pi^0 \pi^0 \gamma$ ,  $K_S K_S K_S \gamma$ ,  $K_S K_S \eta^{(\prime)} \gamma$ , and so on.

Indeed, as long as all the final state particles are eigenstates of charge conjugation, regardless of their spin or other quantum numbers, Eqn. 23 applies since charge conjugation does not map one point of phase space to another. A case of particular interest is  $B^0 \rightarrow K_S \phi \gamma$ . This final state has the practical advantage that the  $\phi$  can be observed as  $K^+ K^-$ , allowing a simple determination of the  $B$  decay vertex, rather than the extrapolation which is necessary when using  $K_S \rightarrow \pi^+ \pi^-$ . Note that there is an indication that this mode has a branching fraction of  $\sim 5 \times 10^{-6}$  [12]. Other analogous cases of interest are  $K_S \rho \gamma$  and  $K_S \omega \gamma$ .

If  $P_1$  and  $P_2$  are not self-conjugate but are anti-particles of each other then the mode of analysis is the same as  $\pi^+ \pi^- \gamma$  above. In this case by considering the time dependent Dalitz plot we can separately determine  $\phi^q$  and  $\psi^q$ . Final states of this sort are  $\pi^+ \pi^- \gamma$  and  $K^+ K^- \gamma$ . Table I shows the various final states which may result from  $B_d$  and  $B_s$  decay and which quark-level process they are sensitive to.

Let us now consider a few of these cases which are likely to be of greatest experimental interest. In the case of  $B_d$  decay preliminary studies have been done of the AGS mode  $K_S \pi^0 \gamma$  on the  $K^*$  resonance [7]. Part of the importance of the discussion in this paper is that this data may be combined with nonresonant  $K_S \pi^0 \gamma$  decays and also with signal from other resonances. Since we are interested in the underlying two-body process  $b \rightarrow q\gamma$ , we assume that experimental cuts will be imposed to discriminate against bremsstrahlung and other possible background. At a  $B$ -factory experiment, these cuts will typically include a requirement on the center-of-mass energy of the photon; since the  $B$  meson is almost at rest in the  $\Upsilon(4S)$  rest frame this is equivalent to requiring a hard photon and will remove most bremsstrahlung events. Further cuts may include other requirements on the  $P_1 P_2 \gamma$  phase space. In addition to reducing the dominant experimental background from continuum  $e^+ e^- \rightarrow q\overline{q}$ ,  $q = u, d, s, c$  events, these requirements can remove the background caused by two body hadronic  $B$  decays followed by radiative hadronic decays. For example, the decay chain  $B_d \rightarrow \omega K_S$ ,  $\omega \rightarrow \pi^0 \gamma$ , contributes to the final state  $K_S \pi^0 \gamma$ . This background decay has a product branching fraction of  $2.5 \times 10^{-7}$  [23], which is small, but not entirely negligible compared to that for the signal. However, this background can be removed with a requirement on the invariant mass of the  $\pi^0 \gamma$  combination. Similar backgrounds should be considered for each final state.

Replacing the  $\pi^0$  with an  $\eta^{(\prime)}$  is more challenging experimentally, although the decay  $B \rightarrow K \eta \gamma$  has recently been

observed [32]. These decays should measure the same quantity if the decay is controlled by Eqn. 2. It would however be of some importance to use these modes as a check that new physics proceeds through this dipole operator.

In the  $B_d$  system  $K_S K_S \gamma$  monitors  $\sin \phi^d \sin 2\psi^d$  for the  $b \rightarrow d\gamma$  transition. Of course this case may also be subject to a significant amount of direct  $CP$  violation at the quark level which can also be measured in the usual way in a time (in)dependent analysis, and could be further checked via charged  $B$  decays such as  $B^+ \rightarrow K^+ K_S \gamma$  or  $B^+ \rightarrow \pi^+ \pi^0 \gamma$ .

The  $P^+ P^- \gamma$  states  $B_d \rightarrow \pi^+ \pi^- \gamma$  and  $B_d \rightarrow K^+ K^- \gamma$  could in principle be used to separately obtain  $\phi^d$  and  $\psi^d$ . The  $B_d \rightarrow K^+ K^- \gamma$  case is, however, OZI suppressed and so is unlikely to have a significant branching ratio. The same is true for  $B_s \rightarrow \pi^+ \pi^- \gamma$ . Experimentally, good  $K/\pi$  separation is needed to isolate  $B_d \rightarrow \pi^+ \pi^- \gamma$  from the decay  $B_d \rightarrow K^+ \pi^- \gamma$  which may have an order of magnitude greater branching ratio.

Observing  $B_s$  oscillations at hadronic  $B$  machines will of course be challenging. The simplest of the modes in Table I to measure is probably  $B_s \rightarrow K^+ K^- \gamma$  which again is sensitive to the  $b \rightarrow s\gamma$  transition.

## VII. ANNIHILATION CONTRIBUTION

The key feature of oscillations in  $B_q \rightarrow F\gamma$  is that they only may take place if  $\psi \neq 0$ . In the SM, this reaction takes place through the penguin in Fig. 1. The photon due to this penguin process is exactly left handed in the limit of massless  $q$ ; since in that case  $q$  would be left handed, so conservation of helicity would imply that the photon would likewise necessarily be left handed. More generally, in the SM this process gives rise to  $\psi \approx m_q/m_b$ .

Another source of right handed photons which could therefore potentially produce a signal is the annihilation diagram shown in Fig. 4. This process would only contribute as a background to be present in  $b \rightarrow d\gamma$  processes in  $B_d$  decays and to  $b \rightarrow s\gamma$  processes in  $B_s$  decays. These processes are subject to large nonperturbative effects when the photon is radiated colinearly with the initial light quark. This enhancement, however, is only in the left handed photon channel. To see this consider the amplitude of the relevant annihilation graph (d) where we have applied a Fierz transformation to the  $W$  propagator:

$$\mathcal{M}_{\text{ann}} \propto \frac{1}{p_d^2 - 2q \cdot p_d} (\bar{d} \not{E} (\not{q} - \not{p}_d) \gamma^\mu L b) (\bar{d} \gamma_\mu L u), \quad (38)$$

where  $p_d$  is the momentum of the initial  $d$  quark,  $L = (1 - \gamma^5)/2$  and  $R = (1 + \gamma^5)/2$ . We can rewrite this in the following form:

$$\mathcal{M}_{\text{ann}} \propto -i \frac{f_B}{p_d^2 - 2q \cdot p_d} (\bar{d} [\sigma^{\mu\nu} E_\mu q_\nu R] \gamma^\mu b) (\bar{d} \gamma_\mu L u) + \frac{f_B}{p_d^2 - 2q \cdot p_d} (\bar{d} \not{E} \not{p}_d \gamma^\mu L b) (\bar{d} \gamma_\mu L u). \quad (39)$$

where  $f_B$  is the  $B$  decay constant and  $E_\mu$  is the polarization 4-vector of the photon. The first term is by itself gauge invariant. The factor in square brackets is the dipole operator for the emission of left handed photons. This term is enhanced by the propagator in front because this quantity in the rest frame of the  $B$  meson is approximately  $1/(E_\gamma E_d)$  where  $E_d$  is expected to be small  $\approx \lambda_{QCD}$ . As discussed in [19] this term corresponds to emission of a colinear photon by a light (initial) quark and because of its singular nature in perturbation theory it is expected to receive non-perturbative corrections. Since it has the same photon helicity as the penguin produced photons but a different CKM phase, it will alter the magnitude and phase of  $F_L^q$  but will not contribute to  $F_R^q$  and therefore will not much affect  $\psi^q$ .

Actually, the singular nature of the first term in Eqn. 39 is a consequence of the very simple non-relativistic model used for the bound state. Indeed in more sophisticated discussions, that quantity is not singular anymore but has a well defined light-cone expansion in  $1/E_\gamma$  [20]; the important point relevant to this paper is that the left-handed nature of the simplistic picture above survives the improved theoretical treatment.

The second term is not gauge invariant but must form a gauge invariant set when added to the other bremsstrahlung diagrams. This term is proportional to the 4-momentum of the light quark and so the energy of the light quark in the numerator will tend to cancel the denominator. Therefore, though these graphs will produce photons of both helicity they are not enhanced by a light quark propagator and so are expected to make only a negligible contribution.

Thus we conclude that the annihilation contribution does not cause any serious difficulty to the analysis above.

## VIII. NON-DIPOLE OPERATORS

A potential complication for the use of this method as a test for new physics is the SM contribution to radiative decays through non-dipole operators [33], for instance, the penguin  $b \rightarrow s\gamma g$ . Such processes generally do not fix the

helicity of the photon and so would result in a SM contribution to  $\mathcal{S}$ . Also the precise contribution of such processes to particular final states is difficult to calculate reliably. Fortunately, the experiments being suggested here can give a handle on the extent of these non-dipole contributions and the presence of the latter need not represent a serious limitation to the application of our method to search of NP.

The different operator structure in  $H_{\text{eff}}$  would mean, that in contrast to the pure dipole case,  $\mathcal{S}$  would depend on the kinematics and composition of the final state. For example, for the case of modes such as  $K_S\pi^0(\eta, \eta')\gamma$  (see the discussion after Eqns. 23 and 24) the presence of non-dipole contributions would, in general, make the asymmetry  $\mathcal{S}$  depend on the Dalitz variables  $s_1$  and  $z$ . Thus, the contamination from non-dipole terms in  $H_{\text{eff}}$  may be estimated by fitting the experimental data on  $\mathcal{S}$  to a suitable parametrization of the dependence on  $s_1$  and/or  $z$ . A difference in the values of  $\mathcal{S}$  for two resonances of identical  $J^{PC}$  would also indicate non-dipole contributions. Similarly for the  $\pi^+\pi^-\gamma$  mode,  $\psi$  and  $\phi$  would depend on  $s_1$  and  $z$ . For a better understanding of these non-leading effects, it is also useful to measure  $\mathcal{S}$  for a number of different final states and kinematic ranges and see if there is variation. For instance, in the case of  $b \rightarrow s\gamma$  some decays which should be studied are those suggested above of the form  $B^0 \rightarrow \gamma K_S + \text{neutrals}$  where neutrals could include  $\pi^0, \eta^{(\prime)}, \rho^0, \omega, \phi, K_S, \dots$  or any combination of such mesons.

## IX. SUMMARY

We have extended the work of [6] so that data from other resonances as well as nonresonant contributions to final states such as  $K_S\pi^0\gamma, K_S f_0\gamma, K^+K^-\gamma, \pi^+\pi^-\gamma, K_S\eta^{(\prime)}\gamma$  and  $K_SK_S\gamma$  can be included for the  $b \rightarrow s$  or  $b \rightarrow d$  transitions relevant to  $B_d$  or  $B_s$  decays. This should significantly improve the effectiveness of testing the Standard Model with mixing-induced  $CP$  violation in radiative  $B$  decays. Indeed, this not only helps in reducing the statistical errors, since no separation between resonant and nonresonant events is needed, it should also help in reducing the systematic errors. For the  $K_S\pi^0\gamma$  type of final state an improved determination of the product  $\sin 2\psi^q \sin \phi^q$ , where  $\phi^q$  is the weak phase and  $\psi^q$  monitors the photon helicity, is possible. For final states such as  $\pi^+\pi^-\gamma$  and  $K^+K^-\gamma$ , separate determination of each of these two quantities is possible.

A key feature of these modes is that the dipole interaction in  $H_{\text{eff}}$  gives rise to a mixing-induced  $CP$  asymmetry which is independent of the Dalitz variables. Recall that in many models of New Physics it is the (dimension five) dipole interaction that is likely to be dominant. Thus the study of the  $CP$  asymmetries of these modes should be useful in searching for New Physics even in the presence of non-dipole SM contributions.

In passing, we briefly recall the hierarchy of  $CP$  asymmetries in radiative  $B$  decays expected in the Standard Model. Assuming the dipole Hamiltonian dominates, for  $b \rightarrow s$  the mixing-induced  $CP$  asymmetry is expected to be  $O(3\%)$  and the direct  $CP$  asymmetry [5] should be around 0.6%. For  $b \rightarrow d$ , the direct  $CP$  asymmetry is expected to be around 15% whereas the mixing-induced  $CP$  asymmetry is  $\approx 0.1\%$ , making it into a very interesting (essentially) *null test* of the SM.

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## Appendix: The Effect of a Perturbative Phase

As discussed in [5] the  $b \rightarrow d\gamma$  transition is expected to receive contributions with different weak and strong phases which could lead to appreciable direct  $CP$  violation at the quark level. If we take this effect into account in the case of  $B_d \rightarrow \pi^+\pi^-\gamma$ , the modified values of  $X, Y$  and  $Z$  are:

$$\begin{aligned} X(\pi^+\pi^-\gamma) &= F^2 \left\{ [|g(s_1, z)|^2 + |g(s_1, -z)|^2] (1 + \delta^2) + 2\delta \cos^2 \psi [\cos(\mu - \nu)g(s_1, z) + \cos(\mu + \nu)g(s_1, -z)] \right\}, \\ Y(\pi^+\pi^-\gamma) &= F^2 \left\{ [|g(s_1, z)|^2 - |g(s_1, -z)|^2] (1 + 2\delta \cos^2 \psi \cos \mu \cos \nu) - \right. \\ &\quad \left. [|g(s_1, z)|^2 + |g(s_1, -z)|^2] 2\delta \cos^2 \psi \sin \mu \sin \nu \right\}, \quad (40) \\ Z(\pi^+\pi^-\gamma) &= -2(F^d)^2 \left\{ \text{Re}(g(s_1, -z)g^*(s_1, z)) [\sin \phi^d + \delta \cos \mu \sin(\phi^d + \nu)] + \right. \\ &\quad \left. \text{Im}(g(s_1, -z)g^*(s_1, z)) [\cos \phi^d \delta \cos \mu \cos(\phi^d + \nu)] \right\} \sin 2\psi^d. \end{aligned}$$

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  - [25] Note that the definition of the  $CP$  asymmetry used here follows the convention of [22] and is opposite to that used in [6]; see also [24].
  - [26] We use a definition  $q/p \equiv e^{+i\phi_M}$  in this paper, while the opposite sign (*i.e.*  $q/p \equiv e^{-i\phi_M}$ ) is taken in [6].
  - [27] Note another change from [6]; in that paper it was stated that  $CP$  of the resonance  $V$  ( $K^*$  or  $\rho$ ) determines the sign of oscillations; in practice for  $V$  case it makes no essential difference.
  - [28] Throughout our theoretical discussion,  $\pi^0$  can always be replaced by  $\eta$  or  $\eta'$  or any other non-strange scalar.
  - [29] We have used a reference frame where the  $B$  is at rest with the decay occurring in the  $xz$  plane, the photon momentum along the  $-z$  axis and the  $x$  component of the  $K_S$  momentum is positive. If we use a more general coordinate system where the photon momentum is in the  $-z$  direction, then we need to introduce the azimuthal angle,  $\phi$ , of the  $K_S$  momentum. In this case,  $\mathcal{M}_{\bar{L}}(s_1, z) = \pm F_{\bar{L}}^* f(s_1, z) e^{\pm i\phi}$ .
  - [30] This conclusion differs from the very brief discussion regarding three body modes given in the second para of p.188 of [6].
  - [31] As in the previous case, we have used a reference frame where the  $B$  is at rest with the decay occurring in the  $xz$  plane, the photon momentum along the  $-z$  axis and the  $x$  component of the  $\pi^+$  momentum is positive. In the more general case where the photon momentum is in the  $-z$  direction, then we need to introduce the azimuthal angle,  $\phi$ , of the  $\pi^+$ . In this case,  $\overline{\mathcal{M}}_{\bar{L}}(s_1, z) = \pm F_{\bar{L}}^* g(s_1, z) e^{\pm i\phi}$ . Note however that the application of charge conjugation on the phase space  $(z, \phi)$  in Eqn. 4 gives  $\mathcal{M}_{\bar{L}}(s_1, z) = \pm F_{\bar{L}}^* g(s_1, -z) e^{\pm i(\phi+\pi)} C_{\text{internal}} = \pm F_{\bar{L}}^* g(s_1, -z) e^{\pm i\phi}$  since  $C_{\text{internal}} = C_\gamma = -1$  while the change  $\phi \rightarrow \phi + \pi$  due to interchange of  $\pi^+ \leftrightarrow \pi^-$  also introduces a factor of  $(-1)$ .
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